Date:

Subject: Lec 2

Classify

① $u_x^2 + u u_y = 1$

$O = 1$ $d = 2$ C: non Linear

② $u_t + u u_x + u_{xxx} = 1$

$O = 3$ $d = 1$ C: Almost Linear

③ $u_{xxx} + u u_{xxyy} + u_{yyy} = 0$

$O = 4$ $d = 1$ C: quasi Linear

④ $u u_x - 2xy u_y = 0$

$O = 1$ $d = 1$ C: quasi Linear

* Give an Example For almost First order PDE.

Not Existed

$u_x u_y = 1$ non Linear

$u_x u_y + u = 1$ non Linear

The general Solution of PDE

$u(x) \rightarrow u_x = 1 \rightarrow u = x + c$

(O.D.E) ك ثابت عددي

$u(x,y) \rightarrow u_x = 1 \rightarrow u = x + f(y) + c$

ثابت التكامل

الناتج من تكامل u_x

$u = x + y + c$

$u = x + \sin y + c$

$u = x + e^y + c$

$u = x + \sin e^y \cos y + c$

$u \in R^3 \rightarrow u(x,y,z) \rightarrow u_x = 1$

$u = x + f(y) + f(z) + c$

$u \in R^3 * T \rightarrow u(x,y,z,t) \rightarrow u_x = 1$

$u = x + f(y) + g(z) + h(t) + c$

Ex: Find the general Solution of the PDE

① $\frac{\partial^2 u}{\partial x \partial y} = 0 \rightarrow u_{xy} = 0$

$u(x,y) \in R^3$

$$\frac{\partial u_x}{\partial y} = 0$$

$$u_x = f(x) + c$$

$$\frac{\partial u}{\partial x} = f(x) + c$$

$$\int \partial u = \int f(x) \partial x + c \partial x$$

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$$\boxed{u(x,y) = F(x) + \underline{G(y)}}$$

ثابت بتكامل

F & G are orbtirory Function

② $u_{xyz} = 0$

$$u(x,y,z) = F(x) + G(y) + h(z)$$

احد الحلول

$u_{xyz} = 0 \qquad \dfrac{\partial u_{xy}}{\partial z} = 0$

$u_{xy} = F(x) + G(y) \qquad \& \quad f(x)\, g(y)$

$$\frac{\partial u_x}{\partial y} = F(x) + G(y)$$

$$u_x = F(x)\, y + \int G(y)\, dy + h(x) + J(z)$$

$$\frac{\partial u}{\partial x} = F(x)y + \int G(y)\, dy + h(x) + J(z)$$

$$u = \int F(x)\, y\, \partial x + \int G(y)\, dy\, \partial x + \int h(x)\, \partial x + \int J(z)\, \partial x + K(y) + L(z)$$

$$u(x,y,z) = F(x) + G(y) + h(z) + x[P(y)] + y[Q(x)] + z[R(x)]$$

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The general Solution of PDE using Direct integration التفاضل المباشر

Ex: Find the general Solution of the Following PDE:

* $u_{xyzt} = 0$

① $u_{xy} = 0$

$$u = F(x) + g(y)$$

② $u_{xy} = y^2 + 1$

تكامل بالنسبة ل y

$$u_x = \frac{y^3}{3} + y + F(x)$$

اكامل بالنسبة ل x

$$u = \frac{y^3}{3} x + yx + \int F(x)\,dx + G(y)$$

$$= \frac{xy^3}{3} + xy + F(x) + G(y)$$

③ $u_{xy} + u_y = 1$

$u_x + u = y + f(x)$ ثابت التكامل بالنسبة ل y

$$\frac{\partial u}{\partial x} + u = y + F(x)$$

O.D.E $\rightarrow$ Linear

$P(x) = 1$ $\quad$ $Q(x) = y + F(x)$

$\mu = e^{\int P(x)\,dx}$ $\quad$ $\mu = e^x$

$\square = \int \mu\, Q(x)\,dx + \text{Constant}$

$\square\, u(x,y) = \int e^x [y + F(x)\,dx + G(y)]$

ثابت التكامل بالنسبة ل x

General Sol.

How to Verify the Solution of PDE

the Solution of PDE should be Satisfied this PDE لابد ان يحقق المعادلة التفاضلية الجزئية

Ex: Verify that the Functions.

$u(x,y) = x^2 - y^2$

$u(x,y) = e^y \sin y$

$u(x,y) = 2xy$

are Solution of the eqn

$u_{xx} + u_{yy} = 0$

افاضل مرتين بالنسبة ل x

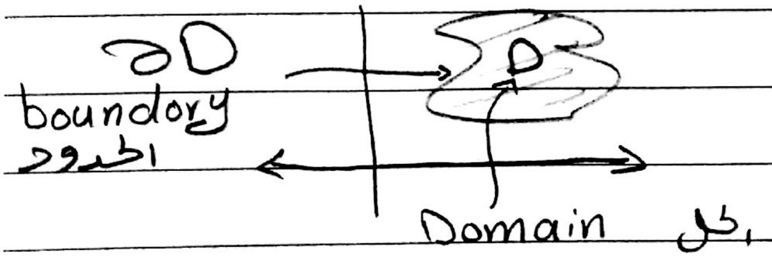
" " " ل y

اعوض ..

① $2 - 2 = 0$ Satisfy

② $e^x \sin y - e^x \sin y = 0$ Satisfy

③ $u_{xx} = 0 \quad u_{yy} = 0$ Satisfy



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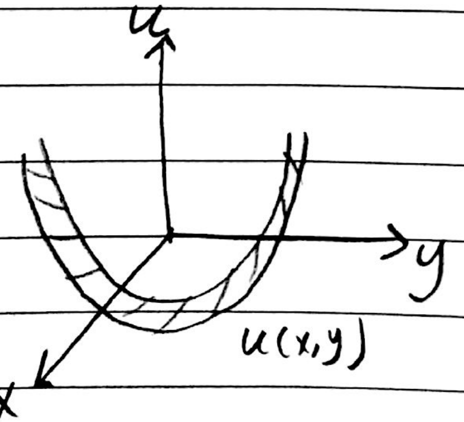

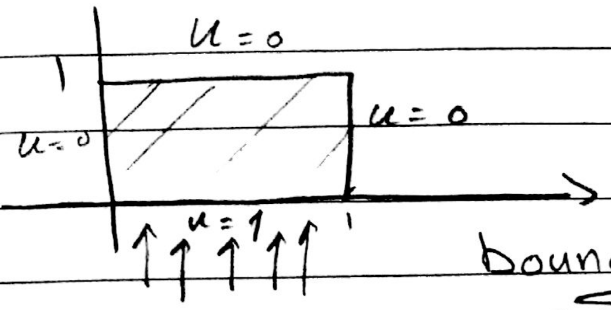

يختار الحل الذي يحقق ال boundary